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Parameter perturbation method for dynamic responses of structures with uncertain-but-bounded parameters based on interval analysis

Zhiping Qiu *, Xiaojun Wang

Institute of Solid Mechanics, Beijing University of Aeronautics and Astronautics, Beijing 100083, PR China

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Abstract

The study was intended to evaluate the range of dynamic responses of structures with uncertain-but-bounded parameters by using the parameter perturbation method. The uncertain parameters were modeled as an interval vector. The first-order perturbation quantities of responses of the perturbed system were obtained through the parameter perturbation method, and then taking advantage of interval mathematics a new algorithm to estimate the response interval was presented. Comparisons between the parameter perturbation method and the probabilistic approach from mathematical proofs and numerical simulations were performed. The numerical results are in agreement with the mathematical proofs. The response range given by the parameter perturbation method encloses that obtained by the probabilistic approach. The results also show good robustness of the proposed method.

Keywords: Dynamic response; Parameter perturbation; Uncertain-but-bounded parameters; Interval mathematics; Probabilistic approach; Finite element analysis

1. Introduction

Dynamic response analysis of structures plays an important role in the design of structural systems. However, due to manufacturing errors, measurement errors and other factors, the structural geometric properties and mechanical properties are usually uncertain. As a result, the structural dynamic response

* Corresponding author. Tel.: +86 10 8231 7507; fax: +86 10 8232 8501.
E-mail address: zpqiu@buaa.edu.cn (Z. Qiu).

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is also uncertain. Therefore, it is important to estimate the effect of these uncertainties on the structural dynamic response.

It is customary to assume that the given system has statistics properties in determining the response of a dynamic system with uncertainty. Astill et al. (1972) modeled the uncertain parameters as random variables and calculated the response of structures by use of the Monte Carlo method. A great deal of computational efforts is needed especially for the MDOF system despite it is accurate. A probabilistic approach to evaluate the effect of uncertainties in geometrical and material properties of structures on the vibration response of random excitation was presented by Chen et al. (1992). If the statistics properties of uncertain parameters are known it will be an effective method. Unfortunately, the sufficient prior knowledge about the uncertain structural parameters is often absent or wrongly assumed; thus, the probabilistic results may be invalid. In practice, only the bounds on their amplitude are often known. Therefore, in recent years some non-probabilistic approaches, such as convex models (Ben-Haim and Elishakoff, 1990; Ben-Haim, 1994; Ben-Haim et al., 1996; Qiu, 2003) and interval analysis methods (Qiu and Elishakoff, 1998; Qiu and Wang, 2003), are developed to deal with the bounded uncertainties.

On the basis of the parameter perturbation method and interval mathematics, a new algorithm to evaluate the dynamic response range of structures with uncertainties is presented in this paper. Comparisons between the presented method and the probabilistic approach from two aspects of mathematics proofs and numerical simulations are carried out.

2. Problem statement

Consider the equation of motion (Meirovotch, 1980; Weaver and Johnston, 1987) of a general dynamic system with n degrees of freedom in the following form

$$\boldsymbol{M}\ddot{\boldsymbol{x}}(t) + \boldsymbol{C}\dot{\boldsymbol{x}}(t) + \boldsymbol{K}\boldsymbol{x}(t) = \boldsymbol{F}(t) \tag{1}$$

where $\boldsymbol{M} = (m_{ij})$, $\boldsymbol{C} = (c_{ij})$ and $\boldsymbol{K} = (k_{ij})$ are the mass, damping and stiffness matrices; $\boldsymbol{F}(t) = (f_i(t))$ is the external load vector. $\boldsymbol{x}(t) = (x_i(t))$, $\dot{\boldsymbol{x}}(t) = (\dot{x}_i(t))$ and $\ddot{\boldsymbol{x}}(t) = (\ddot{x}_i(t))$ are the displacement, velocity, and acceleration vectors of the finite element assemblage, respectively. $\boldsymbol{M} = (m_{ij})$ is the positive definite matrix. $\boldsymbol{C} = (c_{ij})$ and $\boldsymbol{K} = (k_{ij})$ are the positive semi-definite matrices.

By the finite element analysis, we know that the mass matrix $\boldsymbol{M} = (m_{ij})$, the damping matrix $\boldsymbol{C} = (c_{ij})$, the stiffness matrix $\boldsymbol{K} = (k_{ij})$ and the external load vector $\boldsymbol{F}(t) = (f_i(t))$ often depend on the structural parameter vector $\boldsymbol{a} = (a_i)$, and may be expressed as their functions, i.e.

$$\boldsymbol{M} = \boldsymbol{M}(\boldsymbol{a}) = (m_{ij}(\boldsymbol{a})), \quad \boldsymbol{C} = \boldsymbol{C}(\boldsymbol{a}) = (c_{ij}(\boldsymbol{a})) \tag{2a}$$

$$\boldsymbol{K} = \boldsymbol{K}(\boldsymbol{a}) = (k_{ij}(\boldsymbol{a})), \quad \boldsymbol{F}(t) = \boldsymbol{F}(\boldsymbol{a}, t) = (f_i(\boldsymbol{a}, t)) \tag{2b}$$

in which $\boldsymbol{a} = (a_i)$ is an m-dimensional vector. Thus, Eq. (1) can be rewritten as

$$\boldsymbol{M}(\boldsymbol{a})\ddot{\boldsymbol{x}}(\boldsymbol{a}, t) + \boldsymbol{C}(\boldsymbol{a})\dot{\boldsymbol{x}}(\boldsymbol{a}, t) + \boldsymbol{K}(\boldsymbol{a})\boldsymbol{x}(\boldsymbol{a}, t) = \boldsymbol{F}(\boldsymbol{a}, t) \tag{3}$$

Consider a realistic situation in which the available information on the structural parameter vector $\boldsymbol{a} = (a_i)$ is not enough to justify an assumption on its probabilistic characteristics, we follow the thought of interval mathematics or interval analysis (Moore, 1979; Alefeld and Herzberger, 1983) and assume that the structural parameter vector $\boldsymbol{a} = (a_i)$ belongs to a bounded convex set—interval vector

$$\boldsymbol{a} \in \boldsymbol{a}^I = [\underline{\boldsymbol{a}}, \bar{\boldsymbol{a}}] = (a_i^I), \quad a_i \in a_i^I = [\underline{a}_i, \bar{a}_i], \quad i = 1, 2, \ldots, m \tag{4}$$

where $\bar{\boldsymbol{a}} = (\bar{a}_i)$ and $\underline{\boldsymbol{a}} = (\underline{a}_i)$ are the upper and lower bounds of structural parameters $\boldsymbol{a} = (a_i)$, respectively. From interval mathematics, we know that Eq. (4) describes a "box" with m order of dimension.

Suppose that the upper bound vector $\bar{\boldsymbol{a}} = (\bar{a}_i)$ and the lower bound vector $\underline{\boldsymbol{a}} = (\underline{a}_i)$ of the structural parameter vector $\boldsymbol{a} = (a_i)$ are given, the objective is to find all the possible dynamic responses $\boldsymbol{x}(t)$ satisfying the dynamic equation (3), where $\boldsymbol{a}$ is assumed all possible values inside the interval parameter vector $\boldsymbol{a}^I$. This infinite number of dynamic responses constitute a bounded response set

$$\boldsymbol{\Gamma} = \left\{\boldsymbol{x}(\boldsymbol{a},t) : \boldsymbol{M}(\boldsymbol{a})\ddot{\boldsymbol{x}}(\boldsymbol{a},t) + \boldsymbol{C}(\boldsymbol{a})\dot{\boldsymbol{x}}(\boldsymbol{a},t) + \boldsymbol{K}(\boldsymbol{a})\boldsymbol{x}(\boldsymbol{a},t) = \boldsymbol{F}(\boldsymbol{a},t),\ \boldsymbol{a} \in \boldsymbol{a}^I\right\} \tag{5}$$

In general, the set $\boldsymbol{\Gamma}$ has a very complicated region.

In interval mathematics (Moore, 1979; Alefeld and Herzberger, 1983), solving the dynamic response problem (3) subject to (4) is synonymous to find a multi-dimensional rectangle or interval vector containing dynamic response set (5) for the interval structural parameter vector. In other words, we seek the upper and lower bounds (or interval dynamic response vector) on the dynamic response set (5), i.e.

$$\boldsymbol{x}^I(\boldsymbol{a},t) = [\underline{\boldsymbol{x}}(\boldsymbol{a},t), \bar{\boldsymbol{x}}(\boldsymbol{a},t)] = (x_i^I(\boldsymbol{a},t)) \tag{6}$$

where $\bar{\boldsymbol{x}}(\boldsymbol{a},t) = (\bar{x}_i(\boldsymbol{a},t))$ and $\underline{\boldsymbol{x}}(\boldsymbol{a},t) = (\underline{x}_i(\boldsymbol{a},t))$, and

$$\bar{\boldsymbol{x}}(\boldsymbol{a},t) = \max\{\boldsymbol{x}(\boldsymbol{a},t) : \boldsymbol{x}(\boldsymbol{a},t) \in R^n, \boldsymbol{M}(\boldsymbol{a})\ddot{\boldsymbol{x}}(\boldsymbol{a},t) + \boldsymbol{C}(\boldsymbol{a})\dot{\boldsymbol{x}}(\boldsymbol{a},t) + \boldsymbol{K}(\boldsymbol{a})\boldsymbol{x}(\boldsymbol{a},t) = \boldsymbol{F}(\boldsymbol{a},t),\ \boldsymbol{a} \in \boldsymbol{a}^I\} \tag{7}$$

and

$$\underline{\boldsymbol{x}}(\boldsymbol{a},t) = \min\{\boldsymbol{x}(\boldsymbol{a},t) : \boldsymbol{x}(\boldsymbol{a},t) \in R^n, \boldsymbol{M}(\boldsymbol{a})\ddot{\boldsymbol{x}}(\boldsymbol{a},t) + \boldsymbol{C}(\boldsymbol{a})\dot{\boldsymbol{x}}(\boldsymbol{a},t) + \boldsymbol{K}(\boldsymbol{a})\boldsymbol{x}(\boldsymbol{a},t) = \boldsymbol{F}(\boldsymbol{a},t),\ \boldsymbol{a} \in \boldsymbol{a}^I\} \tag{8}$$

In the sequel, our aim is to determine the upper and lower bounds of the interval dynamic response.

3. First-order parameter perturbation of the structural dynamic response problem

In structural dynamics, a frequently encountered problem is how to take into account in analysis design change introduced after the structural dynamics analysis has been completed and the dynamic responses have been computed. If the new design is drastically different from the old one, then a completely new analysis and computational cycle is very necessary. But if the new design varies only slightly different from the old one, then the problem is whether the information from the old design can be used to extract information concerning the new design. In particular, the problem of interest here is whether the dynamic response solution already available can be used to derive the dynamic response corresponding to the new data, without extensive additional computations. In order to solve the structural dynamic response problem, in the sequel, we introduce the matrix perturbation technique.

Let us consider the n-dimensional structural dynamic system $\boldsymbol{M}_0$, $\boldsymbol{C}_0$, $\boldsymbol{K}_0$ and $\boldsymbol{F}_0(t)$, and denote its dynamic responses by $\ddot{\boldsymbol{x}}_0(t)$, $\dot{\boldsymbol{x}}_0(t)$ and $\boldsymbol{x}_0(t)$, where the dynamic responses satisfy

$$\boldsymbol{M}_0\ddot{\boldsymbol{x}}_0(t) + \boldsymbol{C}_0\dot{\boldsymbol{x}}_0(t) + \boldsymbol{K}_0\boldsymbol{x}_0(t) = \boldsymbol{F}_0(t) \tag{9}$$

Next consider the structural dynamic response problem associated with the n-dimensional structural dynamic system

$$\boldsymbol{M} = \boldsymbol{M}_0 + \boldsymbol{\delta M} \tag{10}$$

and

$$\boldsymbol{C} = \boldsymbol{C}_0 + \boldsymbol{\delta C} \tag{11}$$

and

$$\boldsymbol{K} = \boldsymbol{K}_0 + \boldsymbol{\delta K} \tag{12}$$

and

$$\boldsymbol{F}(t) = \boldsymbol{F}_0(t) + \boldsymbol{\delta F}(t) \tag{13}$$

where the first terms on the right hand sides of Eqs. (10)–(13) $\boldsymbol{M}_0$, $\boldsymbol{C}_0$, $\boldsymbol{K}_0$ and $\boldsymbol{F}_0(t)$ are the original dynamic system and the second terms $\boldsymbol{\delta M}$, $\boldsymbol{\delta C}$, $\boldsymbol{\delta K}$ and $\boldsymbol{\delta F}(t)$ represent small changes from $\boldsymbol{M}_0$, $\boldsymbol{C}_0$, $\boldsymbol{K}_0$ and $\boldsymbol{F}_0(t)$. We shall refer to $\boldsymbol{M}$, $\boldsymbol{C}$, $\boldsymbol{K}$ and $\boldsymbol{F}(t)$ as the perturbed system and to $\boldsymbol{\delta M}$, $\boldsymbol{\delta C}$, $\boldsymbol{\delta K}$ and $\boldsymbol{\delta F}(t)$ as the first-order perturbation system. By contrast, $\boldsymbol{M}_0$, $\boldsymbol{C}_0$, $\boldsymbol{K}_0$ and $\boldsymbol{F}_0(t)$ represent the unperturbed system. The perturbed structural dynamic response problem can be written in the form

$$\boldsymbol{M}\ddot{\boldsymbol{x}}(t) + \boldsymbol{C}\dot{\boldsymbol{x}}(t) + \boldsymbol{K}\boldsymbol{x}(t) = \boldsymbol{F}(t) \tag{14}$$

where $\boldsymbol{x}(t) = (x_i(t))$, $\dot{\boldsymbol{x}}(t) = (\dot{x}_i(t))$ and $\ddot{\boldsymbol{x}}(t) = (\ddot{x}_i(t))$ are, respectively, the perturbed dynamic displacement, velocity and acceleration vectors.

Our interest lies in the first-order perturbation dynamic responses. Because $\boldsymbol{M}$, $\boldsymbol{C}$, $\boldsymbol{K}$ and $\boldsymbol{F}(t)$ are obtained from $\boldsymbol{M}_0$, $\boldsymbol{C}_0$, $\boldsymbol{K}_0$ and $\boldsymbol{F}_0(t)$ through small perturbations, it follows that as long as the system is stable the perturbed dynamic displacement, velocity and acceleration vectors can be written in the form

$$\boldsymbol{x}(t) = \boldsymbol{x}_0(t) + \boldsymbol{\delta x}, \quad \dot{\boldsymbol{x}}(t) = \dot{\boldsymbol{x}}_0(t) + \boldsymbol{\delta}\dot{\boldsymbol{x}}, \quad \ddot{\boldsymbol{x}}(t) = \ddot{\boldsymbol{x}}_0(t) + \boldsymbol{\delta}\ddot{\boldsymbol{x}} \tag{15}$$

where $\boldsymbol{\delta x} = (\delta x_i)$, $\boldsymbol{\delta}\dot{\boldsymbol{x}} = (\delta\dot{x}_i)$ and $\boldsymbol{\delta}\ddot{\boldsymbol{x}} = (\delta\ddot{x}_i)$ are the first-order perturbations. Substituting Eqs. (10)–(13), (15) into Eq. (14), we can obtain

$$(\boldsymbol{M}_0 + \boldsymbol{\delta M})(\ddot{\boldsymbol{x}}_0(t) + \boldsymbol{\delta}\ddot{\boldsymbol{x}}) + (\boldsymbol{C}_0 + \boldsymbol{\delta C})(\dot{\boldsymbol{x}}_0(t) + \boldsymbol{\delta}\dot{\boldsymbol{x}}) + (\boldsymbol{K}_0 + \boldsymbol{\delta K})(\boldsymbol{x}_0(t) + \boldsymbol{\delta x}) = \boldsymbol{F}_0(t) + \boldsymbol{\delta F}(t) \tag{16}$$

The following problem is the determination of the perturbations $\boldsymbol{\delta x} = (\delta x_i)$, $\boldsymbol{\delta}\dot{\boldsymbol{x}} = (\delta\dot{x}_i)$ and $\boldsymbol{\delta}\ddot{\boldsymbol{x}} = (\delta\ddot{x}_i)$ based on the assumption that $\boldsymbol{M}_0$, $\boldsymbol{C}_0$, $\boldsymbol{K}_0$, $\boldsymbol{F}_0(t)$, $\boldsymbol{\delta M}$, $\boldsymbol{\delta C}$, $\boldsymbol{\delta K}$, $\boldsymbol{\delta F}(t)$, $\ddot{\boldsymbol{x}}_0(t)$, $\dot{\boldsymbol{x}}_0(t)$ and $\boldsymbol{x}_0(t)$ are known. Here, we will concentrate on the determination of $\boldsymbol{\delta x} = (\delta x_i)$.

We will seek a first-order approximate solution, so that the second-order terms in Eq. (16) will be ignored. Expanding Eq. (16) and combining Eq. (9) yield

$$\boldsymbol{M}_0\ddot{\boldsymbol{x}}_0(t) + \boldsymbol{C}_0\dot{\boldsymbol{x}}_0(t) + \boldsymbol{K}_0\boldsymbol{x}_0(t) = \boldsymbol{F}_0(t) \tag{17}$$

$$\boldsymbol{M}_0\boldsymbol{\delta}\ddot{\boldsymbol{x}}(t) + \boldsymbol{C}_0\boldsymbol{\delta}\dot{\boldsymbol{x}}(t) + \boldsymbol{K}_0\boldsymbol{\delta x}(t) = \boldsymbol{\delta F}(t) - (\boldsymbol{\delta M}\ddot{\boldsymbol{x}}_0(t) + \boldsymbol{\delta C}\dot{\boldsymbol{x}}_0(t) + \boldsymbol{\delta K}\boldsymbol{x}_0(t)) \tag{18}$$

Decomposing the mass matrix $\boldsymbol{M} = (m_{ij})$, the damping matrix $\boldsymbol{C} = (c_{ij})$, the stiffness matrix $\boldsymbol{K} = (k_{ij})$ and the external load vector $\boldsymbol{F}(t) = (f_i(t))$ with respect to the structural parameter vector $\boldsymbol{a} = (a_j) \in R^m$ can result in

$$\boldsymbol{M}(\boldsymbol{a}) = \sum_{j=1}^{m} a_j M_j = a_1 M_1 + a_2 M_2 + \cdots + a_m M_m \tag{19}$$

and

$$\boldsymbol{C}(\boldsymbol{a}) = \sum_{j=1}^{m} a_j C_j = a_1 C_1 + a_2 C_2 + \cdots + a_m C_m \tag{20}$$

and

$$\boldsymbol{K}(\boldsymbol{a}) = \sum_{j=1}^{m} a_j K_j = a_1 K_1 + a_2 K_2 + \cdots + a_m K_m \tag{21}$$

and

$$\boldsymbol{F}(\boldsymbol{a}, t) = \sum_{j=1}^{m} a_j F_j(t) = a_1 F_1(t) + a_2 F_2(t) + \cdots + a_m F_m(t) \tag{22}$$

where M_j, C_j, K_j and $F_j(t)$ are, respectively, the mass matrix, the damping matrix, the stiffness matrix and the external load vector associated with the structural parameter a_j. This decomposition is generally called the non-negative decomposition of a matrix. Such decompositions arise naturally in a practical engineering context. For example, in structural finite element analysis, M_i, C_i, and K_i may be taken as the element mass and stiffness matrices (or possibly substructure matrices) corresponding to the structural parameter a_j.

Let $\boldsymbol{a}_0 = (a_{0_j}) \in R^m$ be the nominal value of the structural parameter vector $\boldsymbol{a} = (a_j) \in R^m$ which might be visualized as the average value of the structural parameter vector $\boldsymbol{a} = (a_j) \in R^m$. Then, the structural parameter vector slightly different from the nominal values $\boldsymbol{a}_0 = (a_{0j}) \in R^m$ could be denoted by

$$\boldsymbol{a} = \boldsymbol{a}_0 + \boldsymbol{\delta} \tag{23}$$

in which $\boldsymbol{\delta} = (\delta_j) \in R^m$ is small quantity. Thus, Eq. (15) can be rewritten as

$$\boldsymbol{x}(t) = \boldsymbol{x}(\boldsymbol{a}, t) = \boldsymbol{x}(\boldsymbol{a}_0 + \boldsymbol{\delta}, t) = \boldsymbol{x}(\boldsymbol{a}_0, t) + \boldsymbol{\delta x}(\boldsymbol{a}, t) = \boldsymbol{x}_0(t) + \boldsymbol{\delta x}(\boldsymbol{a}, t) \tag{24}$$

where

$$\boldsymbol{x}_0(t) = (x_{0i}(t)), \quad \boldsymbol{\delta x}(\boldsymbol{a}, t) = (\delta x_i(\boldsymbol{a}, t)) \tag{25}$$

It follows that Eqs. (17) and (18) are also represented as

$$\boldsymbol{M}_0\ddot{\boldsymbol{x}}_0(\boldsymbol{a}, t) + \boldsymbol{C}_0\dot{\boldsymbol{x}}_0(\boldsymbol{a}, t) + \boldsymbol{K}_0\boldsymbol{x}_0(\boldsymbol{a}, t) = \boldsymbol{F}_0(\boldsymbol{a}, t) \tag{26}$$

and

$$\boldsymbol{M}_0\boldsymbol{\delta}\ddot{\boldsymbol{x}}(\boldsymbol{a}, t) + \boldsymbol{C}_0\boldsymbol{\delta}\dot{\boldsymbol{x}}(\boldsymbol{a}, t) + \boldsymbol{K}_0\boldsymbol{\delta x}(\boldsymbol{a}, t) = \boldsymbol{\delta F}(\boldsymbol{a}, t) - (\boldsymbol{\delta M}\ddot{\boldsymbol{x}}_0(t) + \boldsymbol{\delta C}\dot{\boldsymbol{x}}_0(t) + \boldsymbol{\delta K x}_0(t)) \tag{27}$$

Obviously, the structural dynamic responses $\ddot{\boldsymbol{x}}_0(t)$, $\dot{\boldsymbol{x}}_0(t)$ and $\boldsymbol{x}_0(t)$ of the unperturbed structural dynamic system can be calculated from Eq. (26) by using the common method, such as Wilson-θ. In order to obtain the first-order perturbations of the perturbed structural dynamic system from Eq. (27), by means of the perturbation method, we assume that

$$\boldsymbol{\delta x}(\boldsymbol{a}, t) = \sum_{j=1}^{m} \delta_j \boldsymbol{X_j}(t) = \sum_{j=1}^{m} \boldsymbol{X_j}\delta_j \tag{28}$$

where

$$\boldsymbol{\delta x}(\boldsymbol{a}, t) = (\delta x_i(\boldsymbol{a}, t)), \quad \boldsymbol{X_j} = (X_{ij}), \quad j = 1, 2, \ldots, m \tag{29}$$

Taking the first and the second derivatives of Eq. (28) with respect to time t yields

$$\boldsymbol{\delta}\dot{\boldsymbol{x}}(\boldsymbol{a}, t) = (\delta\dot{x}_i(\boldsymbol{a}, t)) = \sum_{j=1}^{m} \delta_j \dot{\boldsymbol{X}}_{\boldsymbol{j}}(t) = \sum_{j=1}^{m} \dot{\boldsymbol{X}}_{\boldsymbol{j}}\delta_j = \left(\sum_{j=1}^{m} \dot{X}_{ij}\delta_j\right) \tag{30}$$

and

$$\boldsymbol{\delta}\ddot{\boldsymbol{x}}(\boldsymbol{a}, t) = (\delta\ddot{x}_i(\boldsymbol{a}, t)) = \sum_{j=1}^{m} \delta_j \ddot{\boldsymbol{X}}_{\boldsymbol{j}}(t) = \sum_{j=1}^{m} \ddot{\boldsymbol{X}}_{\boldsymbol{j}}\delta_j = \left(\sum_{j=1}^{m} \ddot{X}_{ij}\delta_j\right) \tag{31}$$

Substituting Eq. (23) into Eq. (19) yields

$$\begin{aligned}\boldsymbol{M}(\boldsymbol{a}) &= \boldsymbol{M}(\boldsymbol{a}_0 + \boldsymbol{\delta}) = \sum_{j=1}^{m} a_j \boldsymbol{M_j} = \sum_{j=1}^{m} (a_{0j} + \delta_j)\boldsymbol{M_j} = \sum_{j=1}^{m} (a_{0j}\boldsymbol{M_j} + \delta_j\boldsymbol{M_j}) \\ &= \sum_{j=1}^{m} a_{0j}\boldsymbol{M_j} + \sum_{j=1}^{m} \delta_j\boldsymbol{M_j} = \boldsymbol{M}_0 + \boldsymbol{\delta M}\end{aligned} \tag{32}$$

where

$$\boldsymbol{M}_0 = \sum_{j=1}^{m} a_{0j}\boldsymbol{M_j}, \quad \boldsymbol{\delta M} = \sum_{j=1}^{m} \delta_j \boldsymbol{M_j} = \sum_{j=1}^{m} \boldsymbol{M_j}\delta_j \tag{33}$$

Similarly

$$\boldsymbol{C}(\boldsymbol{a}) = \boldsymbol{C}(\boldsymbol{a}_0 + \boldsymbol{\delta}) = \boldsymbol{C_0} + \boldsymbol{\delta C} \tag{34}$$

where

$$\boldsymbol{C}_0 = \sum_{j=1}^{m} a_{0j}\boldsymbol{C_j}, \boldsymbol{\delta C} = \sum_{j=1}^{m} \delta_j \boldsymbol{C_j} = \sum_{j=1}^{m} \boldsymbol{C_j}\delta_j \tag{35}$$

and

$$\boldsymbol{K}(\boldsymbol{a}) = \boldsymbol{K}(\boldsymbol{a}_0 + \boldsymbol{\delta}) = \boldsymbol{K_0} + \boldsymbol{\delta K} \tag{36}$$

where

$$\boldsymbol{K}_0 = \sum_{j=1}^{m} a_{0j}\boldsymbol{K_j}, \quad \boldsymbol{\delta K} = \sum_{j=1}^{m} \delta_j \boldsymbol{K_j} = \sum_{j=1}^{m} \boldsymbol{K_j}\delta_j \tag{37}$$

and

$$\boldsymbol{F}(\boldsymbol{a}, t) = \boldsymbol{F}(\boldsymbol{a}_0 + \boldsymbol{\delta}, t) = \boldsymbol{F}_0(t) + \boldsymbol{\delta F}(t) \tag{38}$$

where

$$\boldsymbol{F}_0(t) = \sum_{j=1}^{m} a_{0j}\boldsymbol{F_j}(t), \quad \boldsymbol{\delta F}(t) = \sum_{j=1}^{m} \delta_j \boldsymbol{F_j}(t) = \sum_{j=1}^{m} \delta_j \boldsymbol{F_j} = \sum_{j=1}^{m} \boldsymbol{F_j}\delta_j \tag{39}$$

Substituting Eqs. (28), (30), (31), (33), (35), (37) and (39) into Eq. (27) yields

$$\sum_{j=1}^{m} (\boldsymbol{M}_0\ddot{\boldsymbol{X}}_{\boldsymbol{j}} + \boldsymbol{C}_0\dot{\boldsymbol{X}}_{\boldsymbol{j}} + \boldsymbol{K}_0\boldsymbol{X}_{\boldsymbol{j}})\delta_j = \sum_{j=1}^{m} (\boldsymbol{F_j} - (\boldsymbol{M_j}\ddot{\boldsymbol{x}}_0(t) + \boldsymbol{C_j}\dot{\boldsymbol{x}}_0(t) + \boldsymbol{K_j}\boldsymbol{x}_0(t)))\delta_j \tag{40}$$

Comparing the coefficients of the perturbation parameter δ_j, $j = 1, 2, \ldots, m$, we obtain

$$\boldsymbol{M}_0\ddot{\boldsymbol{X}}_{\boldsymbol{j}} + \boldsymbol{C}_0\dot{\boldsymbol{X}}_{\boldsymbol{j}} + \boldsymbol{K}_0\boldsymbol{X}_{\boldsymbol{j}} = \boldsymbol{F_j} - (\boldsymbol{M_j}\ddot{\boldsymbol{x}}_0(t) + \boldsymbol{C_j}\dot{\boldsymbol{x}}_0(t) + \boldsymbol{K_j}\boldsymbol{x}_0(t)), \quad j = 1, 2, \ldots, m \tag{41}$$

Therefore, the solution of Eq. (27) is now transformed into the solution of Eq. (41). Because there is no uncertain variables in Eq. (41), it is convenient to solve Eq. (41) similar to Eq. (26).

4. Interval analysis method

In this section, we will calculate the interval dynamic response vector of structures with uncertain-but-bounded parameters by use of interval mathematics.

By means of Eq. (4), we may define the nominal value vector or midpoint vector (Moore, 1979; Alefeld and Herzberger, 1983) of the interval structural parameter vector as

$$\boldsymbol{a}_0 = (a_{0i}) = m(\boldsymbol{a}^I) = \frac{(\bar{\boldsymbol{a}} + \underline{\boldsymbol{a}})}{2}, \quad a_{0i} = m(a_i^I) = \frac{(\bar{a}_i + \underline{a}_i)}{2}, \quad i = 1, 2, \ldots, m \tag{42}$$

and the deviation amplitude vector or the uncertain radius vector of the interval structural parameter vector as

$$\Delta\boldsymbol{a} = (\Delta a_i) = \mathrm{rad}(\boldsymbol{a}^I) = \frac{(\bar{\boldsymbol{a}} - \underline{\boldsymbol{a}})}{2}, \quad \Delta a_i = \mathrm{rad}(a_i^I) = \frac{(\bar{a}_i - \underline{a}_i)}{2}, \quad i = 1, 2, \ldots, m \tag{43}$$

Thus, based on interval mathematics, the interval structural parameter vector is decomposed into the sum of the nominal value vector and the deviation vector, i.e.

$$\boldsymbol{a}^I = [\underline{\boldsymbol{a}}, \bar{\boldsymbol{a}}] = [\boldsymbol{a}_0 - \Delta\boldsymbol{a}, \boldsymbol{a}_0 + \Delta\boldsymbol{a}] = [\boldsymbol{a}_0, \boldsymbol{a}_0] + [-\Delta\boldsymbol{a}, \Delta\boldsymbol{a}] = \boldsymbol{a}_0 + \Delta\boldsymbol{a}^I = \boldsymbol{a}_0 + \Delta\boldsymbol{a}[-1, 1] = \boldsymbol{a}_0 + \Delta\boldsymbol{a}e_\Delta \tag{44}$$

where $\bar{\boldsymbol{a}} = \boldsymbol{a}_0 + \Delta\boldsymbol{a}$, $\underline{\boldsymbol{a}} = \boldsymbol{a}_0 - \Delta\boldsymbol{a}$, $\Delta\boldsymbol{a}^I = [-\Delta\boldsymbol{a}, \Delta\boldsymbol{a}]$, $e_\Delta = [-1, 1]$.

In terms of Eq. (44), the interval structural parameter vector may be written in the following form

$$\boldsymbol{a} = \boldsymbol{a}_0 + \boldsymbol{\delta}, \quad |\boldsymbol{\delta}| \leqslant \Delta\boldsymbol{a} \tag{45}$$

Combining Eq. (24) with (28), the dynamic response $\boldsymbol{x}(\boldsymbol{a}, t) = (x_i(\boldsymbol{a}, t))$ can be written as

$$\boldsymbol{x}(\boldsymbol{a}, t) = \boldsymbol{x}(\boldsymbol{a}_0 + \boldsymbol{\delta}, t) = \boldsymbol{x}(\boldsymbol{a}_0, t) + \boldsymbol{\delta x}(\boldsymbol{a}, t) = \boldsymbol{x}_0 + \sum_{j=1}^{m} \boldsymbol{X_j}\delta_j \tag{46}$$

where

$$\delta_j \in \Delta a_j^I = [-\Delta a_j, \Delta a_j], \quad j = 1, 2, \ldots, m \tag{47}$$

By making use of the natural interval extension in interval mathematics, from Eq. (46), we can obtain the interval vector of the dynamic responses of structures

$$\boldsymbol{x}^I(\boldsymbol{a}, t) = \boldsymbol{x}(\boldsymbol{a}_0, t) + \sum_{j=1}^{m} |\boldsymbol{X_j}|\Delta a_j^I \tag{48}$$

where $|\cdot|$ denotes absolute value componentwise.

After the interval operations, from the above equation, we have

$$\bar{\boldsymbol{x}}(\boldsymbol{a}, t) = \boldsymbol{x}(\boldsymbol{a}_0, t) + \sum_{j=1}^{m} |\boldsymbol{X_j}|\Delta a_j \tag{49a}$$

or component forms

$$\bar{\boldsymbol{x}}_i(\boldsymbol{a}, t) = \boldsymbol{x}_i(\boldsymbol{a}_0, t) + \sum_{j=1}^{m} |\boldsymbol{X}_{ij}|\Delta a_j, \quad i = 1, 2, \ldots, n \tag{49b}$$

and

$$\underline{\boldsymbol{x}}(\boldsymbol{a}, t) = \boldsymbol{x}(\boldsymbol{a}_0, t) - \sum_{j=1}^{m} |\boldsymbol{X_j}|\Delta a_j \tag{50a}$$

or component forms

$$\underline{x}_i(\boldsymbol{a}, t) = \boldsymbol{x}_i(\boldsymbol{a}_0, t) - \sum_{j=1}^{m} |\boldsymbol{X}_{ij}|\Delta a_j, \quad i = 1, 2, \ldots, n \tag{50b}$$

From Eqs. (49) and (50) we can determine the interval regions of the dynamic responses of structures with uncertain-but-bounded parameters by combining the parameter perturbation method and interval mathematics.

5. Probabilistic approach

In this section, we will determine the interval dynamic response of structures with uncertain-but-bounded parameters by the probabilistic approach.

Assume that the m-dimensional uncertain structural parameter vector $\boldsymbol{a} = (a_i)$ is random variable (Elishakoff, 1983). Thus, the dynamic response $\boldsymbol{x}(\boldsymbol{a}, t)$ is also random. If we denote the random structural parameter vector's expected value, or the mean value (MV), by

$$E(\boldsymbol{a}) = (E(a_i)) = \boldsymbol{a}^E = (a_i^E) \tag{51}$$

then Eq. (46) can be interpreted as the first order perturbation of the random dynamic response about the mean value $x_i(\boldsymbol{a}^E, t)$, $i = 1, 2, \ldots, n$ of the random structural parameter vector $\boldsymbol{a} = (a_i)$.

For the random structural parameter vector $\boldsymbol{a} = (a_i)$, the variance is defined by

$$\mathrm{Var}(\boldsymbol{a}) = (\mathrm{Var}(a_i)) = D(\boldsymbol{a}) = (D(a_i)) \tag{52}$$

Then the standard deviation of the random structural parameter vector $\boldsymbol{a} = (a_i)$ is defined as

$$\sigma(\boldsymbol{a}) = (\sigma(a_i)) = \sqrt{\mathrm{Var}(\boldsymbol{a})} = \sqrt{D(\boldsymbol{a})} = (\mathrm{Var}(a_i)) = (\sqrt{D(a_i)}) \tag{53}$$

In terms of Eq. (51), the mean value or expected value of the dynamic response is obtained by taking the expected value of both side of Eq. (46). In so doing, it follows that

$$E\{\boldsymbol{x}(\boldsymbol{a}, t)\} = E\{\boldsymbol{x}(\boldsymbol{a}^E, t)\} + E\left(\sum_{j=1}^{m} \boldsymbol{X_j}\delta_j\right) = \boldsymbol{x}(\boldsymbol{a}^E, t) + \sum_{j=1}^{m} \boldsymbol{X_j} E(a_j - a_j^E) \tag{54}$$

and noting that the term $E(\delta_j) = E(a_j - a_j^E)$ is zero, we obtain

$$E\{x_i(\boldsymbol{a}, t)\} = x_i(\boldsymbol{a}^E, t), \quad i = 1, 2, \ldots, n \tag{55}$$

For the variance of the dynamic response $\boldsymbol{x}(\boldsymbol{a}, t) = (x_i(\boldsymbol{a}, t))$, in a similar way we can obtain as follows

$$\mathrm{Var}(\boldsymbol{x}(\boldsymbol{a}, t)) = D(\boldsymbol{x}(\boldsymbol{a}, t)) = \sum_{j=1}^{m} (\boldsymbol{X}_j *' \boldsymbol{X}_j) D(a_j) = \sum_{k=1}^{m} \sum_{l=1}^{m} (\boldsymbol{X_k} *' \boldsymbol{X_l}) \mathrm{Cov}(a_k, a_l) \tag{56}$$

where $*'$ denotes component multiplication and generates a vector; $\mathrm{Cov}(a_k, a_l)$ is the covariance of the random structural parameter variables and is defined as

$$\mathrm{Cov}(a_k, a_l) = E[(a_k - E[a_k])(a_l - E[a_l])] \tag{57}$$

When the random structural parameter variables are independent, the variance of the dynamic response can be reduced as

$$\mathrm{Var}(x_i(\boldsymbol{a}, t)) = D(x_i(\boldsymbol{a}, t)) = \sum_{j=1}^{m} (X_{ij})^2 D(a_j) = \sum_{j=1}^{m} (X_{ij}\sigma(a_j))^2 = \sum_{j=1}^{m} (X_{ij}\sigma_j)^2 \tag{58}$$

Obviously, the standard deviation of the dynamic response $x(\boldsymbol{a},$ txtcommenta$t)$ is

$$\sigma(\boldsymbol{x}(\boldsymbol{a}, t)) = \sqrt{D(\boldsymbol{x}(\boldsymbol{a}, t))} = \sqrt{\sum_{j=1}^{m} (\boldsymbol{X_j} *' \boldsymbol{X}_j)\sigma_j^2} \tag{59}$$

Thus, let k be a positive integer, the probabilistic region of k times standard deviations of its mean value of the random dynamic response is

$$\boldsymbol{y}^I = [\underline{\boldsymbol{y}}(\boldsymbol{a}, t), \overline{\boldsymbol{y}}(\boldsymbol{a}, t)] = [\boldsymbol{x}(\boldsymbol{a}^E, t) - k\sigma(\boldsymbol{x}(\boldsymbol{a}, t)), \boldsymbol{x}(\boldsymbol{a}^E, t) + k\sigma(\boldsymbol{x}(\boldsymbol{a}, t))] \tag{60a}$$

or component form

$$\boldsymbol{y}_i^I = [\underline{\boldsymbol{y}}_i(\boldsymbol{a},t), \bar{\boldsymbol{y}}_i(\boldsymbol{a},t)] = [\boldsymbol{x}_i(\boldsymbol{a}^E,t) - k\sigma(\boldsymbol{x}_i(\boldsymbol{a},t)), \boldsymbol{x}_i(\boldsymbol{a}^E,t) + k\sigma(\boldsymbol{x}_i(\boldsymbol{a},t))] \tag{60b}$$

where the probabilistic upper bound is

$$\bar{\boldsymbol{y}}_i(\boldsymbol{a},t) = \boldsymbol{x}_i(\boldsymbol{a}^E,t) + k\sigma(\boldsymbol{x}_i(\boldsymbol{a},t)) = \boldsymbol{x}_i(\boldsymbol{a}^E,t) + k\sqrt{\sum_{j=1}^{m}(\boldsymbol{X}_{ij}\sigma_j)^2}, \quad i = 1,2,\ldots,n \tag{61}$$

and the probabilistic lower bound is

$$\underline{\boldsymbol{y}}_i(\boldsymbol{a},t) = \boldsymbol{x}_i(\boldsymbol{a}^E,t) - k\sigma(\boldsymbol{x}_i(\boldsymbol{a},t)) = \boldsymbol{x}_i(\boldsymbol{a}^E,t) - k\sqrt{\sum_{j=1}^{m}(\boldsymbol{X}_{ij}\sigma_j)^2}, \quad i = 1,2,\ldots,n \tag{62}$$

From Eqs. (61) and (62) we can obtain the interval region of the dynamic response of structures with uncertain-but-bounded parameters by using the probabilistic approach. According to the Tchebycheff's inequality, we know that the probability of the random variable with finite variance falling within k standard deviations of its mean is at least $1 - 1/k^2$, and the bound is independent of the distribution of the random variable, provided that it has a finite variance. For sufficient large k, when using the probabilistic approach to estimate the upper and lower bound of structural response, the value of k times standard deviations in Eqs. (61) and (62) will result in almost a certain event.

6. Comparison of non-probabilistic interval analysis method and probabilistic approach

For any real m-tuples $a_i \geqslant 0, 1, 2, \ldots, m$, according to Cauchy–Schwarz inequality the following inequality holds

$$\sum_{i=1}^{m} a_i \geqslant \sqrt{\sum_{i=1}^{m} a_i^2} \tag{63}$$

In the following we will perform a comparison between non-probabilistic interval analysis method and probabilistic approach based on Eq. (63).

Assume that we obtain the interval regions of the uncertain-but-bounded structural parameters based on the probabilistic statistical information or stochastic sample test and they can be expressed as the following interval vector form

$$\boldsymbol{a}^I = [\underline{\boldsymbol{a}}, \bar{\boldsymbol{a}}] = (a_i^I) = [\boldsymbol{a}^E - k\boldsymbol{\sigma}, a^E + k\boldsymbol{\sigma}] \tag{64}$$

where $\bar{\boldsymbol{a}} = (\bar{a}_i),\ \bar{a}_i = a_i^E + k\sigma_i,\ i = 1,2,\ldots,m$, and $\underline{\boldsymbol{a}} = (\underline{a}_i),\ \underline{a}_i = a_i^E - k\sigma_i,\ i = 1,2,\ldots,m$, are respectively the upper bound vector and the lower bound vector of the interval vector $\boldsymbol{a}^I = [\underline{\boldsymbol{a}}, \bar{\boldsymbol{a}}] = (a_i^I)$, the vectors $\boldsymbol{a}^E = (a_i^E)$ and $\boldsymbol{\sigma} = \sigma(\boldsymbol{a}) = (\sigma(a_i)) = (\sigma_i)$ are respectively the mean value and the standard deviation of the uncertain structural parameter vector $\boldsymbol{a} = (a_i)$, and k is a positive integer. According to the Tchebycheff's inequality in probabilistic theory, we know that the probability of the uncertain structural parameter $\boldsymbol{a} = (a_i)$ with finite variance $\boldsymbol{D} = D(\boldsymbol{a}) = (D_i) = (D(a_i))$ falling within k standard deviations $\boldsymbol{\sigma} = \sqrt{\boldsymbol{D}} = (\sigma_i) = (\sqrt{D_i})$ of its mathematical expectation is at least $1 - 1/k^2$, and the bound is independent of the distribution of the uncertain structural parameter, provided it has a finite variance. Obviously, from Eq. (64), the nominal value vector or midpoint vector of the uncertain structural parameter vector $\boldsymbol{a} = (a_i)$ can be calculated as follows

$$\boldsymbol{a}_0 = (a_{0i}) = m(\boldsymbol{a}^I) = \boldsymbol{a}^E, \quad a_{0i} = m(a_i^I) = a_i^E, \quad i = 1, 2, \ldots, m \tag{65}$$

and the deviation amplitude vector or the uncertain radius vector of the uncertain structural parameter vector $\boldsymbol{a} = (a_i)$ can be determined

$$\Delta\boldsymbol{a} = (\Delta a_i) = \mathrm{rad}(a^I) = k\boldsymbol{\sigma}, \quad \Delta a_i = \mathrm{rad}(a_i^I) = k\sigma_i, \quad i = 1, 2, \ldots, m \tag{66}$$

Thus, in terms of the expressions (65) and (66), the interval regions (49) and (50) of the structural dynamic response can be rewritten as

$$\bar{x}_i(\boldsymbol{a}, t) = x_i(\boldsymbol{a}_0, t) + \sum_{j=1}^{m} |X_{ij}|\Delta a_j = x_i(\boldsymbol{a}^E, t) + \sum_{j=1}^{m} |X_{ij}|k\sigma_j, \quad i = 1, 2, \ldots, n \tag{67}$$

and

$$\underline{x}_i(\boldsymbol{a}, t) = x_i(\boldsymbol{a}_0, t) - \sum_{j=1}^{m} |X_{ij}|\Delta a_j = x_i(\boldsymbol{a}^E, t) - \sum_{j=1}^{m} |X_{ij}|k\sigma_j, \quad i = 1, 2, \ldots, n \tag{68}$$

For the sum expression $\sum_{j=1}^{m} |X_{ij}|k\sigma_j$, by means of the inequality (63), we have that

$$\sum_{j=1}^{m} |X_{ij}k\sigma_j| \geqslant \sqrt{\sum_{j=1}^{m} |(X_{ij}k\sigma_j)^2|} = k\sqrt{\sum_{j=1}^{m} (X_{ij}\sigma_j)^2} \tag{69}$$

Since the inequality (69), from Eqs. (61), (62), (67) and (68), we can deduce

$$\underline{x}_i(\boldsymbol{a}, t) \leqslant \underline{y}_i(\boldsymbol{a}, t) \leqslant \bar{y}_i(\boldsymbol{a}, t) \leqslant \bar{x}_i(\boldsymbol{a}, t), \quad i = 1, 2, \ldots, n \tag{70a}$$

and the vector form

$$\underline{\boldsymbol{x}}(\boldsymbol{a}, t) \leqslant \underline{\boldsymbol{y}}(\boldsymbol{a}, t) \leqslant \bar{\boldsymbol{y}}(\boldsymbol{a}, t) \leqslant \bar{\boldsymbol{x}}(\boldsymbol{a}, t) \tag{70b}$$

The expressions (70) indicates that under the condition of the interval vector of the uncertain parameters determined from the probabilistic information, the width of the dynamic response obtained by the interval analysis method is larger than that by the probabilistic approach for structures with uncertain-but-bounded structural parameters. Namely the lower bounds within interval analysis method are smaller than those predicted by the probabilistic approach, and the upper bounds furnished by the interval analysis method are larger than those yielded by the probabilistic approach. This is just the results which we hope, since according to the definition of probabilistic theory and interval mathematics, the region determined by the interval analysis method should contain that predicted by the probabilistic approach.

7. Numerical example

In order to illustrate the effectiveness of the presented parameter perturbation method for the dynamic responses of the structures with uncertainty, we apply it to a two-dimensional truss with 6 nodes and 10 elements as shown in Fig. 1. Young's modulus of the element material is $E = 2.0 \times 10^{11}$ N/m^2 and the element mass density is $\rho = 7800.0$ kg/m^3. The dimension and boundary conditions of the truss are indicated in Fig. 1. Now it is assumed that there are two harmonic sinusoidal excitations $P_1 = -100\,\mathrm{Sin}(100\pi t)N$ and $P_2 = 200\,\mathrm{Sin}(100\pi t)N$ acting on the node 5 in the vertical direction and the node 6 in the horizontal direction respectively, with the initial conditions $\boldsymbol{x}(t) = 0$ and $\dot{\boldsymbol{x}}(t) = 0$. Due to the manufacture errors or measurement errors, the cross-sectional areas of elements exhibit some uncertainties, and are considered to be uncertain-but-bounded parameters. Their intervals are taken as $A_i^I = [A_i^c - k\beta A_i^c, A_i^c + k\beta A_i^c]$, $i = 1, 2, \ldots, 10$, where

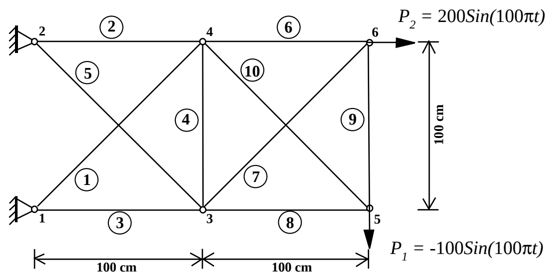

Fig. 1. A 10-bar two-dimensional truss.

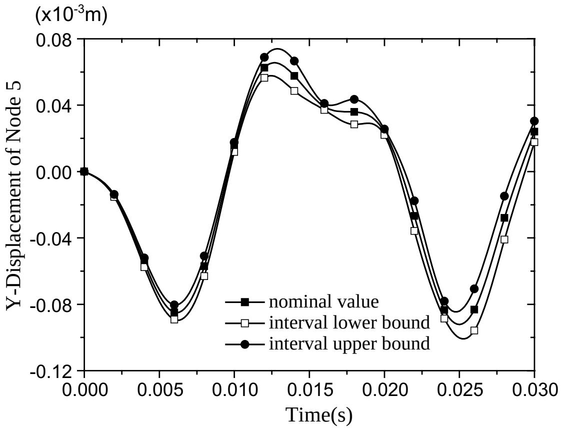

Fig. 2. Response region yielded by the parameter perturbation method.

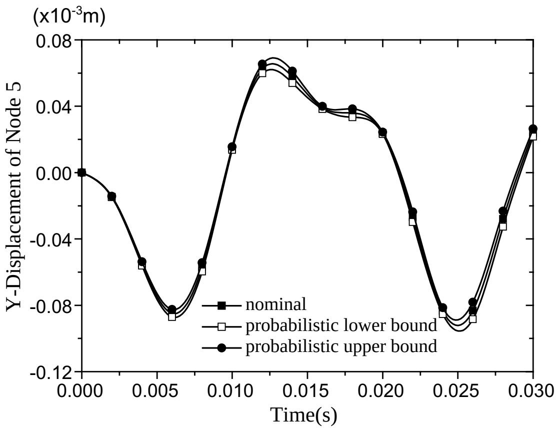

Fig. 3. Response region yielded by the probabilistic approach.

$A_i^c = 1.0 \times 10^{-4}$ m^2, β is a variable coefficient and k is a positive integer. Here β is taken as 0.005, and k is taken as 10. In order to compare with probabilistic approach, these uncertain parameters also are assumed to be random variables with mean value $\mu_{A_i} = A_i^c$ and standard variance $\sigma_{A_i} = \beta A_i^c$. In the following, the displacement responses of the node 5 in the vertical direction are calculated using the presented parameter perturbation method in comparison with the probabilistic approach.

Figs. 2 and 3 describe the response ranges of Node 5 in the vertical direction obtained by the parameter perturbation method and the probabilistic approach, respectively. Comparison between them is plotted in Fig. 4. It can be seen from Figs. 2–4 that the response range by the presented method enclose that by the probabilisitc approach. The numerical results are in agreement with the mathematical proof given in Section 6. Fig. 5 represents the variations of upper and lower bounds of the structural displacement of Node 5 in the vertical direction with uncertain parameter. We can see that when the uncertain parameters are given small change, the variations of the upper and lower bounds of the dynamic response are also small, that is to say, the presented method has a good robustness with respect to uncertainty.

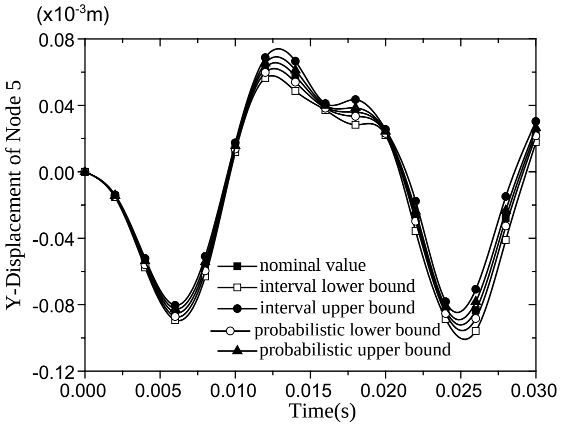

Fig. 4. Comparison of the response region by two methods.

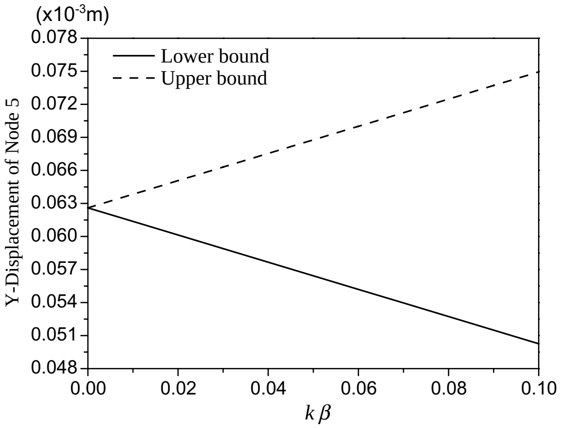

Fig. 5. Variations of upper and lower bounds of the structural response with uncertain parameter by the proposed method ($t = 0.012$ s).

8. Conclusions

In this study, based on the parameter perturbation method and interval mathematics, a new algorithm was presented for determining the range on the dynamic response of structures with uncertain-but-bounded parameters. It need less prior knowledge on uncertain parameters than the probabilistic approach. Comparison between the presented algorithm and the probabilistic approach was performed from two aspects of the mathematical proofs and the numerical simulations. The numerical results are in agreement with the theory proofs, which show that the interval dynamic response obtained by the parameter perturbation method enclose those by the probabilistic approach. Namely the lower bounds within the parameter perturbation method are smaller than those predicted by the probabilistic approach, and the upper bounds furnished by the parameter perturbation method are larger than those yielded by the probabilistic approach. The results also shows that the proposed method has a good robustness with respect to uncertainty.

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